1-1-1978

Estimating the Expected Predictive Accuracy of Econometric Models

Ray C. Fair

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 480

ESTIMATING THE EXPECTED PREDICTIVE ACCURACY OF ECONOMETRIC MODELS

Ray C. Fair

January 17, 1978

ESTIMATING THE EXPECTED PREDICTIVE ACCURACY OF ECONOMETRIC MODELS*

by

Ray C. Fair

I. *Introduction*

A forecast from an econometric model has four main sources of uncertainty. Uncertainty arises from the error terms, the coefficient estimates, the exogenous-variable forecasts, and the possible misspecification of the model.[1] In this paper a method is proposed for estimating the uncertainty from these four sources. Two examples of applying the method are also presented. The first example is for a forecast from the model in Fair [8], and the second example is for a forecast from a naive model. The naive model is meant to be used as a benchmark for the first model. Given the obvious importance of knowing how much confidence to place on any given forecast from a model, it is hoped that the method proposed in this paper will become used by model builders to estimate the degree of uncertainty of their forecasts. A useful by-product of the method is that it also provides a quantitative estimate of the degree of misspecification of a model.

---

*The research described in this paper was financed by grant SOC77-03274 from the National Science Foundation.

[1]Uncertainty also arises from having to make forecasts on the basis of preliminary data rather than the finally-revised data. The uncertainty from this source is probably small relative to the uncertainty from the other four sources, but at any rate it has been ignored in this paper.

The method is outlined in Section II, and its application to the two models is discussed in Sections III-V. The results for the two models are presented and discussed in Section VI. Section VII contains a brief summary of the paper and some concluding remarks.

II. An Outline of the Method

The application of the method requires that a model be estimated and stochastically simulated over a number of different periods. Consider for sake of an example that data for a model are available from quarters 1 through 100,[2] and that the lags in the model are such that the estimation period can begin with quarter 11. Assume that the model has been estimated over the period 11-100. Assume also that a forecast for the period 101-110 has been made using this set of coefficient estimates and some prior forecasts of the exogenous variables for the 101-110 period. This forecast is meant to be dynamic (i.e., generated values of the lagged endogenous variables used after the first quarter), with the actual values through quarter 100 used as initial conditions. The problem is to estimate the degree of uncertainty of this forecast.

If a full-information method has been used to estimate the model, then one has directly available an estimate of the variance-covariance matrix of the error terms and an estimate of the variance-covariance matrix of the coefficient estimates. Otherwise, if only a limited-information method has been used, the former matrix can be estimated using the limited-information estimates of the single-equation residuals,

[2]The data, of course, need not be quarterly. This quarterly assumption is made solely for expositional convenience.

and the latter matrix can be taken to be a block-diagonal matrix, where the blocks are the estimated variance-covariance matrices of the limited-information coefficient estimates of the individual equations. Given these two matrices, it is then straightforward to estimate by means of stochastic simulation the uncertainty due to the error terms and the coefficient estimates. For the above example, each "trial" would be a dynamic simulation for the 101-110 period, with the actual values through quarter 100 used as initial conditions and with the prior forecasts of the exogenous variables for the 101-110 period used for the exogenous-variable values. The exact way in which stochastic simulation was carried out in this study for the two models is described in Sections IV and V.

The uncertainty due to the exogenous-variable forecasts can also be estimated by means of stochastic simulation. This procedure is, however, less straightforward than the above procedure for the error terms and coefficient estimates because there is no obvious estimate available of the degree of uncertainty of the exogenous-variable forecasts themselves. The procedure followed in this study was to regress each exogenous variable in the model on a constant, a linear time trend, and its first eight lagged values, and then to take the estimated standard error from this regression as the estimate of the degree of uncertainty attached to forecasting the change in this variable for each quarter. Given this assumption for each variable and the estimated standard errors, the uncertainty due to the exogenous-variable forecasts could then be estimated by means of stochastic simulation. The exact way in which this stochastic simulation was carried out in this study is also described in Section IV. This regression procedure is not, of course, the only one that can be followed, and all that really needs to be pointed out here in the general outline of the method is that some estimate or assumption about the degree of uncertainty of the exogenous-variable forecasts must be made.

Estimating the uncertainty due to the possible misspecification of the model is the most difficult and costly part of the method. It also rests on one fairly restrictive assumption, namely that the degree of misspecification of the model is (in a sense to be described below) constant across time. This part of the method is best described within the context of the above example, and it is as follows.

Assume first that the above model has been estimated for the period 11-70. Given this set of estimates and given the actual exogenous-variable values for quarter 71, one can estimate by means of stochastic simulation the variances of the one-quarter-ahead forecast errors for the endogenous variables for quarter 71. "Stochastic simulation" for this part of the method means simulation that takes into account the uncertainty from the error terms and the coefficient estimates, but not the exogenous-variable values. For present purposes the exogenous-variable values are always the actual values.

Let y_{i71} denote the actual value of endogenous variable i for quarter 71, let $\bar{y}_{i71}$ denote the (unknown) expected value of y_{i71}, and let σ^2_{i71} denote the (unknown) variance of $y_{i71} - \bar{y}_{i71}$, the forecast error. $\bar{y}_{i71}$ and σ^2_{i71} are, of course, model specific in that they are, among other things, a function of the coefficient estimates of the model. Now, stochastic simulation as outlined above can be used to estimate $\bar{y}_{i71}$ and σ^2_{i71}. Call these estimates $\hat{y}_{i71}$ and $\hat{\sigma}^2_{i71}$. Although it is clearly not the case that these estimates will be exactly equal to the true values, it will be assumed in what follows that the differences between the estimates and the true values are negligible. There are two main reasons the differences are not exactly zero. One is that in any actual use of stochastic simulation only a finite number of draws or trials can be taken. The other, and perhaps more important, reason is that only estimates of the variance-covariance matrices of the error terms and coefficient estimates are available for use in the stochastic simulation. The true variance-covariance matrix of the coefficient estimates, for example, is not known, and so exact stochastic-simulation estimates cannot be achieved even with an infinite number of trials. It is unclear as to the likely size of the differences between the stochastic-simulation estimates and the true values, but, as just mentioned, for present purposes these differences are assumed to be negligible.[3]

Since data on y_{i71} are available, one can compute the actual error in forecasting this variable for quarter 71. The expected or predicted value of y_{i71} is $\hat{\bar{y}}_{i71}$, so that $\hat{\varepsilon}_{i71} = y_{i71} - \hat{\bar{y}}_{i71}$ is the actual forecast error. Now, assuming that $\hat{\bar{y}}_{i71}$ exactly equals $\bar{y}_{i71}$, $\hat{\varepsilon}_{i71}$ is a sample draw from a distribution with a known mean of zero and variance σ^2_{i71}. $\hat{\varepsilon}^2_{i71}$ is thus under this assumption an unbiased estimate of σ^2_{i71}. One thus has two estimates of σ^2_{i71}, one computed from the actual forecast error $(\hat{\varepsilon}^2_{i71})$ and one computed by means of stochastic simulation $(\hat{\sigma}^2_{i71})$. Let d_{i71} denote the difference between these two estimates: $d_{i71} = \hat{\varepsilon}^2_{i71} - \hat{\sigma}^2_{i71}$. Under the assumption that $\hat{\sigma}^2_{i71}$ is exactly equal to the true variance, d_{i71} is the difference between the estimated variance based on the actual forecast error and the true variance. Therefore,

[3]It should also be noted that it is implicitly assumed here that the variances of the forecast errors exist. For some types of coefficient estimates this is not always the case, but no attempt is made in this study to handle this possible problem.

under this assumption and the above assumption that $\hat{\bar{y}}_{i71}$ exactly equals $\bar{y}_{i71}$, the expected value of d_{i71} is zero (since under the above assumption the expected value of $\hat{\varepsilon}_{i71}^2$ is equal to the true variance).

One can repeat the above procedure for quarters 72 through 100. The model can, for example, be reestimated through quarter 71 and the above calculations performed for quarter 72. This will yield a value of d_{i72} for each variable i . Similarly, a value of d_{i73} can be computed by reestimating the model through quarter 72 and performing the above calculations for quarter 73, and so on through quarter 100. This procedure will yield 30 values of d_{it} $(t = 71, 72, \ldots, 100)$ for each variable i , each of the 30 values being based on a different set of coefficient estimates of the model and a different stochastic simulation. If the two assumptions in the previous paragraph hold for all t (i.e., $\hat{\bar{y}}_{it} = \bar{y}_{it}$ and $\hat{\sigma}_{it}^2 = \sigma_{it}^2$ for $t = 71, 72, \ldots, 100$), then the expected value of d_{it} is zero for all t .

The discussion so far is based on the implicit assumption that the model is correctly specified. If the model is not correctly specified, then the expected value of d_{it} will not in general be zero. The stochastic-simulation estimate of the variance of the forecast error for quarter t is based on the assumption that the model is correctly specified for this quarter. The computation of the actual forecast error, on the other hand, is not based on any such assumption. Unlike the stochastic-simulation procedure, the computation of the actual forecast error uses the actual value of y_{it} , and clearly no specification restrictions are imposed on this value.

The key assumption will now be made that the degree of misspecification of the model is constant across time. More precisely, it will be assumed that the expected value of d_{it} is constant across time (i.e., is not a function of t). Although, as noted at the beginning of this section, this is a fairly restrictive assumption, some constancy assumption of this kind is needed before any attempt can be made to estimate the degree of misspecification of a model, and the particular assumption chosen here seemed to be the most natural one to make.[4] Given this assumption, an obvious estimate of the degree of misspecification of a model with respect to the one-quarter-ahead forecast of variable i is the mean of d_{it} in the sample. This estimate, which will be denoted $\bar{d}_i$, is in fact the estimate of the degree of misspecification proposed here. In the above example, $\bar{d}_i$ would be based on a sample size of 30.

Note that $\bar{d}_i$ pertains to the degree of misspecification of a model regarding the one-quarter-ahead forecasts. The above procedure, however, can also be followed for multi-quarter-ahead forecasts. Each length of forecast will have its own $\bar{d}_i$ value, and these values will not in general be the same across lengths. The present procedure thus allows the degree of misspecification of a model to be different for different lengths of forecasts. It should also be noted with respect to the above example that one observation is lost for each one quarter increase in the length of the forecast, given the beginning quarter of 71 and the ending quarter of 100. In other words, 29 values of d_{it} can be computed for the two-quarter-ahead forecasts, 28 values for the three-quarter-ahead forecasts, and so on. In the rest of this paper $\bar{d}_{ik}$ will be used to denote the sample mean of d_{it} for the k-quarter-ahead forecast of variable i .

[4]Note that even given this constancy assumption, the true distribution of d_{it} will depend on the exact way in which the model is misspecified. It thus seems unlikely in any practical application that much could be learned about this distribution.

It should be noted that the expected value of d_{it} may be negative, and so some of the $\bar{d}_{ik}$ values may be negative. Although negative expected values of d_{it} may seem unlikely, it is clearly conceivable for a model to be misspecified in this way. Consider, for example, the case in which the true structure of the economy is changing over time. Any model based on the assumption of a constant structure will be misspecified and will (in a loose sense) have estimated the average structure of the economy over the sample period. If, say, the model has most closely approximated the average structure in the last fourth of the sample period, it may then be the case that the stochastic-simulation estimates of the variances for the period right after the end of the sample period, which are based on the variance-covariance matrices estimated over the entire sample period, will be larger than the variances estimated from the actual forecast errors. Whether this is likely or not is perhaps unclear, but it should at least be noted that negative values of $\bar{d}_{ik}$ are possible.

It should finally be stressed that the $\bar{d}_{ik}$ values are clearly only approximate estimates of the degree of misspecification of a model. The true distribution of d_{it} is not likely to have a mean that is exactly constant across time, as is assumed here. It is an open question how good an approximation this assumption of a constant mean is likely to be.

The method proposed in this paper can now be summarized as follows.

1. Compute values for $\bar{d}_{ik}$. (These computations need not be done each time a new forecast is made since they are not forecast specific.)
2. For a forecast period of, say, quarters 101-110, estimate the model through quarter 100, and then stochastically simulate for the 101-110 period. This stochastic simulation should be based on: (1) the latest

(i.e., through quarter 100) estimated variance-covariance matrices of the error terms and coefficient estimates, (2) actual values through quarter 100 as initial conditions, (3) prior forecasts of the means of the exogenous variables for the 101-110 period, and (4) some estimate or assumption about the degree of uncertainty of the exogenous-variable forecasts. Let $\hat{\sigma}_{it}^2$ denote the stochastic-simulation estimate of the variance of the forecast error for variable i for quarter t . (For $t = 101$ this is a one-quarter-ahead forecast-error variance, for $t = 102$ this is a two-quarter-ahead forecast-error variance, and so on.)

3. Add $\overline{d}_{ik}$ to the appropriate $\hat{\sigma}_{it}^2$. (E.g., add $\overline{d}_{i1}$ to $\hat{\sigma}_{i101}^2$, $\overline{d}_{i2}$ to $\hat{\sigma}_{i102}^2$, and so on.) This sum is then the final estimate of the variance for the forecast error.

This completes the outline of the method. Before proceeding to a discussion of its application to the two models, mention should be made as to how this paper relates to the previous literature. No method similar to the present one appears to have been proposed in the literature. There has in fact been relatively little stochastic simulation of macroeconometric models of any kind. In the following studies stochastic simulation with respect to the error terms only has been performed: Nagar [18] for the condensed version of the Brookings model; Evans, Klein, and Saito [6] for the Wharton model; Fromm, Klein, and Schink [12] for the Brookings model; Green, Liebenberg, and Hirsch [14] for the OBE model; Sowey [19] for the RBA1 model of Australia; Cooper and Fischer [4] for the FMP and St. Louis models; Cooper [3] for the FMP model; Garbade [13] for his model; and Bianchi, Calzolari, and Corsi [1] and Calzolari and Corsi [2] for the ISPE model of Italy. For all these studies except the last two, the number of replications was 50 or less.

There appears to be only one previous study in which stochastic simulation with respect to both the error terms and coefficient estimates was performed. Cooper and Fischer [5] performed nine stochastic-simulation experiments of this type for the St. Louis model, where each experiment was based on 20 replications. Given that the Cooper-Fischer experiments were all within-sample simulations, the present study thus appears to be the first case of the use of stochastic simulation to estimate the outside-sample forecasting accuracy of a macroeconometric model, even with respect to just the error terms and coefficient estimates.

One final comment on the previous literature. It is a common practice to compute root mean squared errors (RMSEs) for econometric models. A typical procedure, for example, is to compute for a particular variable a series of, say, one-quarter-ahead forecast errors, square them, sum the squares, divide the sum by the number of observations, and take the square root of the resulting number. This is the RMSE of the one-quarter-ahead forecast of the variable, and it is typically taken as a measure of the model's forecasting accuracy with respect to this variable. One of the problems with this procedure is that the variances of the forecast errors are not constant across time. They are a function, among other things, of the exogenous-variable values. Although RMSEs are in some loose sense estimates of the averages of the variances across time, no rigorous statistical interpretation can be placed on them. The method proposed in this study does not suffer from this problem: account is always taken of the fact that the variances are not constant across time.

III. The Two Models and Their Forecasts

The two models examined in this study are the model in Fair [8] and a naive model. The naive model is simply one in which each variable is regressed on a constant, a linear time trend, and its first eight lagged values (i.e., each variable y_t is regressed on a constant, t, and $y_{t-1}, \ldots, y_{t-8}$). It is to be used for benchmark purposes. The model in [8] has been changed slightly and updated since [8] was published, and this updated version has been used for purposes of this study. The main change that has been made to the original model is the addition of an equation explaining the behavior of the Federal Reserve. This addition is discussed in Fair [9]. A few definitional changes have also been made to correspond to the 1976 revision of the national income accounts. The updated version of the model consists of 95 equations, 28 of which are stochastic, and has 180 unknown coefficients to estimate (including 14 serial correlation coefficients). The complete list of the equations of this version is presented in [10], which is available from the author upon request.[5] In the following discussion this model will be called Model I, and the naive model will be called Model II.

At the time of this writing (January 1978) I have made two forecasts using Model I, the first issued on July 23, 1977, and the second issued on October 24, 1977. The second forecast is the one whose uncertainty is estimated in this study. For this forecast the model was estimated through 1977II on the basis of data available as of October 1, 1977. (The

---

[5]A tape of the program that solves the model and the latest data set is also available from the author for a cost of $25.00, payable to Yale University. The program is written in FORTRAN-IV and has a user's manual to go with it. With this program one can perform within-sample simulations and make actual outside-sample forecasts.

first quarter of the estimation period was 1954I.) The actual forecast was based on data available as of October 19, 1977, including the preliminary national-income-accounts data for 1977III that were released on this date. The forecast period was 1977IV-1980IV, a total of 13 quarters. For a few variables, primarily exogenous variables, data for 1977III were not available as of October 19, 1977, and for these variables guessed values were used. Guessed values of the exogenous variables for the forecast period also, of course, had to be used. The important hard-to-forecast exogenous variables in the model are the import price index (PIM), the real value of exports (EX), and various fiscal-policy variables. For the fiscal-policy variables, planned government budget numbers were used whenever possible, and for PIM and EX, the simple assumption of a constant future growth rate was used.

The naive model was also used to make a forecast for the 1977IV-1980IV period. The model was estimated through 1977II on the data available as of October 1, 1977,[6] and the actual forecast was based on data available as of October 19, 1977. The naive model has no exogenous variables (except the constant term and the time trend), and so no guessed values of exogenous variables had to be used for this forecast.

The equations of the naive model were estimated by ordinary least squares. The equations of Model I were estimated by two-stage least squares, with account taken, whenever necessary, of first order serial correlation of the error terms. The technique that was used for this purpose is described in Fair [7], and the variables that were used as regressors in

---

[6]The first quarter of the estimation period for the naive model was 1954II. Because of data requirements due to lags, the beginning quarter for the naive model had to be one quarter later than the beginning quarter for Model I.

the first-stage regressions for each equation are listed in [10]. This technique yields an estimate of the first-order serial correlation coefficient for each equation in addition to the estimates of the structural coefficients.

By treating the serial correlation coefficient as a structural coefficient, it is possible to transform an equation with a serially correlated error into an equation without one. This introduces nonlinear restrictions on the coefficients, but otherwise the equation is like any other equation with a non-serially correlated error.[7] Therefore, even though some of the equations of Model I have been estimated under the assumption of first order serial correlation of the error terms, the model should be thought of as a model with nonlinear coefficient restrictions and no serially correlated errors. All references to the variance-covariance matrices of the error terms for Model I, for example, are for the non-serially correlated ("transformed") errors. Likewise, all references to the variance-covariance matrices of the coefficient estimates are for the coefficient estimates *inclusive* of the estimates of the serial correlation coefficients.[8]

---

[7] See, for example, the discussion in Chapter 3 in [8].

[8] In [7], p. 514, I suggested that the variance-covariance matrix of the coefficient estimates inclusive of the estimate of the serial correlation coefficient be estimated by ignoring the correlation between the latter estimate and the other coefficient estimates. Fisher, Cootner, and Baily [11], p. 575, fn. 6, however, have pointed out that one need not ignore this correlation. In computing the estimates of the variance-covariance matrices for use in this study, I have followed the Fisher, Cootner, and Baily advice. In terms of the notation in [7], p. 514, I have estimated the variance-covariance matrix of the coefficient estimates inclusive of the estimate of the serial correlation coefficient as:

$$\hat{\sigma}_{11} \begin{pmatrix} \hat{Q}_1\hat{Q}_1' & \hat{Q}_1\hat{u}_{1_{-1}}' \\ \hat{u}_{1_{-1}}\hat{Q}_1' & \hat{u}_{1_{-1}}\hat{u}_{1_{-1}}' \end{pmatrix}^{-1} .$$

Results for 6 of the 95 endogenous variables of Model I are presented in this study. These six variables are: (1) real GNP, (2) the GNP deflator, (3) the unemployment rate, (4) the wage rate, (5) the bill rate, and (6) the money supply. Results for these six variables are also presented for the naive model. The forecasts of these variables for the 1977IV-1980IV period that are based on setting the error terms in the models equal to zero are presented in the "0" rows in Table 2. Although this procedure of solving models by setting the error terms equal to zero is the one that is followed in practice, it is well known that for a nonlinear model the procedure leads to biased estimates of the true means of the endogenous variables.[9] By comparing the numbers in the 0 rows in Table 2 with the numbers computed by means of stochastic simulation, it is possible to gauge the size of this bias. The results in Table 2 are discussed in Section VI.

IV. Estimated Uncertainty for the Model I Forecast From the Error Terms

Given an estimate of the probability distribution of the error terms in a model, the uncertainty due to these error terms can be estimated by means of stochastic simulation. The exact procedure that was followed in this study in estimating this uncertainty for the Model I forecast is as follows:

1. For the estimation period of 94 observations, 1954I-1977II, consistent estimates of the 28 error terms are available (from the consistent two-stage least squares coefficient estimates). Let E denote the 28×94 matrix of values of these estimated error terms. Given E, the variance-covariance matrix of the 28 error terms was estimated

[9] See, for example, Howrey and Kelejian [17].

as $(1/94)EE'$.[10] Call this matrix S .

2. Let e_t denote the 28×1 vector of values of the error terms for quarter t . The assumption was made that for each t , e_t is normally distributed with mean zero and variance-covariance matrix S :

(1) $$e_t \sim N(0,S) \ , \quad \text{all} \ t \ .$$

3. Given assumption (1), the uncertainty due to the error terms was estimated by means of stochastic simulation. For the forecast period of 13 quarters, each "trial" corresponds to drawing 13 values of e_t (28×13 numbers in all) and computing the forecast using these values.[11] In this first case, where only the uncertainty due to the error terms is being estimated, the coefficient estimates and exogenous-variable

---

[10] Note that EE' was divided by 94, not 94 less some correction for the number of coefficients estimated per equation. No degrees-of-freedom corrections were in fact made for any of the variances and variance-covariance matrices estimated in this study, including those for the naive model. For the naive model, which was estimated by ordinary least squares, there are well defined degrees-of-freedom corrections that could have been made, but this is not the case for Model I. It thus seemed best to put both models on a comparable basis by not adjusting for degrees of freedom in either model. With 94 or 93 observations and at most 10 estimated coefficients per equation, the following results would not have been much different had some adjustment for degrees of freedom been made.

[11] The draws were performed as follows. First, a matrix P was computed such that $PP' = S$. This was done using the LUDECP subroutine in the IMSL library. Then for each of the 13 quarters, 28 values of a standard normal random variable with mean 0 and variance 1 were drawn. This was done using the function RNOR, which is part of the SUPER DUPER random number generator package at Yale. Let u_t denote the 28×1 vector of these draws for quarter t . Then e_t was computed as Pu_t . Since $Eu_tu_t' = I$, then $Ee_te_t' = EPu_tu_t'P' = S$, which is as desired for the distribution of e_t .

values are kept the same for all the trials. The coefficient estimates and exogenous-variable values that were used for this purpose are the same as those used to make the actual forecast.

4. For each trial, one obtains a prediction of each endogenous variable for each quarter. Let y^j_{it} denote the predicted value of variable i for quarter t on the j^{th} trial. If the number of trials is N, then an estimate of the expected value of y_{it} is $(1/N)\sum_{j=1}^{N}\hat{y}^j_{it}$, which will be denoted $\bar{\hat{y}}_{it}$. An estimate of the variance of the forecast error for y_{it} is $(1/N)\sum_{j=1}^{N}(\hat{y}^j_{it} - \bar{\hat{y}}_{it})^2$, which will be denoted $\hat{\sigma}^2_{it}$. The number of trials used for these estimates was 500.[12]

The results of estimating $\hat{\sigma}^2_{it}$ for the six selected variables and the thirteen quarters are presented in the "a" rows of Table 1. For four of the variables, values of $\hat{\sigma}_{it}/\bar{\hat{y}}_{it}$ are presented in the table, and for the other two, values of $\hat{\sigma}_{it}$ are presented. The values of $\bar{\hat{y}}_{it}$ for the six variables and thirteen quarters are presented in the "a" rows of Table 2. These results are discussed in Section VI. Note that estimated standard errors $(\hat{\sigma}_{it})$, not estimated variances $(\hat{\sigma}^2_{it})$, are presented in Table 1.

---

[12] I could see no obvious way to use any of the tricks in, for example, Hammersley and Handscomb [15] to increase the efficiency of the stochastic simulation, and so each trial was merely an independent random draw. Each trial, which consists of solving the model once for 13 quarters, takes about 1.6 seconds on the IBM 370-158 at Yale, so the total time for 500 trials is about 13.3 minutes.

From the Coefficient Estimates

Given an estimate of the probability distribution of the coefficient estimates, the uncertainty due to these estimates can be estimated by means of stochastic simulation. The exact procedure that was followed in this case is as follows:

1. For each of the 28 stochastic equations, an estimate of the variance-covariance matrix of the coefficient estimates is available. Let $\hat{\beta}_i$ denote the vector of coefficient estimates for equation i, let V_i denote the estimated variance-covariance matrix of these estimates, and let β_i^* denote the vector of coefficient values for equation i actually used in a given trial. The assumption was made that β_i^* is normally distributed with mean $\hat{\beta}_i$ and variance-covariance matrix V_i:[13]

$$\beta_i^* \sim N(\hat{\beta}_i, V_i) , \quad i = 1, 2, \ldots, 28 . \tag{2}$$

2. Given assumption (2), the uncertainty due to the coefficient estimates was estimated by means of stochastic simulation. The procedure followed here was actually to estimate the combined uncertainty due to both the error terms and the coefficient estimates. Given this combined estimate for each variable, the uncertainty due only to the coefficient estimates can be estimated as the difference between the combined estimate and the estimate due only to the error terms. In the combined case each trial corresponds to drawing 13 values of e_t (28 x 13 numbers

[13]With reference to the discussion in Section II, assumption (2) means that the variance-covariance matrix of all the coefficient estimates is taken to be block diagonal. Note also, with reference to the discussion in Section III, that the estimates of the serial correlation coefficients are included in the β_i vectors.

in all) *and* a value of β_i^* for each of the 28 equations (180 numbers in all).[14] Aside from drawing 180 extra numbers for each trial, the combined case is the same as the first case. The number of trials used in the combined case was 625.

The results for this case are presented in the "b" rows in Tables 1 and 2.

From the Exogenous-Variable Forecasts

Not counting variables like the constant term, the time trend, and various dummy variables, there are 61 exogenous variables in Model I. The uncertainty due to the fact that these 61 variables must also be forecast was estimated as follows:

1. For the 1954II-1977II period, each of the 61 exogenous variables was regressed on a constant, a linear time trend, and its first eight lagged values. This is the same procedure that was followed for the six endogenous variables of the naive model. For each of these equations, the variance of the error term was estimated as the sum of

---

[14]The draws for the β_i^* vectors were performed as follows. First, for each V_i a matrix P_i was computed such that $P_iP_i' = V_i$. Then for each i , n_i values of a standard normal random variable with mean 0 and variance 1 were drawn, where n_i is the number of coefficients in equation i . Let u_i denote the $n_i \times 1$ vector of these draws. Then β_i^* was computed as $\hat{\beta}_i + P_iu_i$. Since $Eu_iu_i' = I$, then $E(\beta_i^* - \hat{\beta}_i)(\beta_i^* - \hat{\beta}_i)' = EP_iu_iu_i'P_i' = V_i$, which is as desired for the distribution of β_i^* . Subroutine LUDECP and function RNOR were used for these calculations (see footnote 11).

One other point about these calculations should be noted. In the version of Model I used in this study, the coefficient on the lagged dependent variable in the wage-rate equation is constrained to be 0.85. This coefficient clearly has some uncertainty attached to it, and this uncertainty was accounted for in this study by using for V_i for the wage-rate equation the estimated variance-covariance matrix of the unconstrained estimates.

squared residuals divided by the number of observations. Let this estimated variance for equation i be denoted s_i^2 . Also, let u_{it} be a normally distributed random variable with mean zero and variance s_i^2 :

(3) $$u_{it} \sim N(0,\ s_i^2)\ ,\quad \text{all}\quad t\ .$$

2. Regarding the uncertainty in forecasting the exogenous variables, the assumption was made that the error in forecasting the *change* in exogenous variable i from period t-1 to t is u_{it} , where u_{it} is distributed as in (3). Given this assumption, the uncertainty due to the exogenous-variable forecasts can be estimated by stochastic simulation, as will now be described.

3. Let $\hat{x}_{it}$ be the value of exogenous variable i for quarter t that was used for the actual forecast, and let x_{it}^* be the value of x_{it} actually used in a given trial. Also, let r denote the first quarter of the forecast period (1977IV). Then from the assumption in 2, x_{it}^* $(t = r,\ r+1,\ \ldots,\ r+12)$ is:

(4.1) $$x_{ir}^* = \hat{x}_{ir} + u_{ir}$$

(4.2) $$x_{ir+1}^* = \hat{x}_{ir+1} + u_{ir} + u_{ir+1}\ ,$$

$$\vdots$$

(4.13) $$x_{ir+12}^* = \hat{x}_{ir+12} + u_{ir} + u_{ir+1} + \ldots + u_{ir+12}\ .$$

Equation (4.1) states that the value of x_{ir} used in a given trial deviates from the base value by u_{ir} . Equation (4.2), on the other hand, states that the value of x_{ir+1} used in a given trial deviates from the base value by $u_{ir} + u_{ir+1}$. Because of the assumption that the forecast errors pertain to the change in the exogenous variables, the error term u_{ir} is carried along from quarter to quarter. Similarly, u_{ir+1} is carried along from quarter $r+1$ on, and so on.

4. Given equations (4.1)-(4.13), the uncertainty due to the exogenous-variable forecasts was estimated by means of stochastic simulation. Again, the procedure followed here was to estimate the combined uncertainty, where in this case "combined uncertainty" means uncertainty from the error terms, the coefficient estimates, and the exogenous-variable forecasts. In this case each trial corresponds to drawing 28 x 13 numbers for the error terms, 180 numbers for the coefficients, and 61 x 13 values of u_{it} for the exogenous variables.[15] Aside from drawing 61 x 13 extra numbers, this case is the same as the previous case. The number of trials used in this case was also 625.

The results for this case are presented in the "c" rows in Tables 1 and 2. As mentioned in Section II, the assumption in 2 regarding the uncertainty of the exogenous-variable forecasts is clearly not the only possible assumption that one could make. My feeling is that the present procedure probably overestimates the uncertainty in forecasting the fiscal-policy variables (because government-budget data are usually quite useful for this purpose, at least up to about six or eight quarter ahead) and underestimates the uncertainty in forecasting variables like the price of imports and the real value of exports (because, among other things, no account is taken of uncertainty due to the coefficient estimates in

---

[15]Drawing the 61 x 13 values of u_{it} is straightforward. For each i, 13 values of a standard normal random variable with mean 0 and variance 1 are drawn, and then each of these values is multiplied by s_i.

the equations). An alternative procedure to the one used here, which would exacerbate the first problem but alleviate the second, would be to add the 61 exogenous-variable equations to the model and then stochastically simulate this expanded version of the model. This expanded version would be one in which there were no exogenous variables. In future work it may be of interest to try this, although it does seem likely to overstate the uncertainty in the model from the exogenous-variable forecasts.

The assumption that the u_{it} errors pertain to forecasting the change in the exogenous variables perhaps requires some discussion. Given the way that many exogenous variables are forecast, by extrapolating past trends or taking variables to be unchanged from their last observed values, it seems likely that any error made in forecasting the level of a variable in, say, the first quarter will persist throughout the forecast period. If this is true, then the present assumption seems better than the assumption that the u_{it} errors pertain to forecasting the level of the exogenous variables. Again, however, this is clearly not the only assumption that can be made.

From the Possible Misspecification of the Model

The procedure for estimating the degree of misspecification of a model has been discussed in Section II, and this discussion will not be repeated here. For present purposes, values of $\bar{d}_{ik}$ were computed for the one-through eight-quarter-ahead forecasts for each of the 95 endogenous variables in the model. In other words, 95×8 values of $\bar{d}_{ik}$ were computed in all. These computations were based on 33 sets of estimates of the model. For all sets of estimates the first quarter of the sample period was 1954I. The last quarter of the sample period was 1968IV for the first set of estimates, 1969I for the second set of estimates, and so on through 1976IV for the 33rd set of estimates.

For each set of estimates the model was stochastically simulated for eight quarters beginning with the second quarter after the end of the estimation period. The second quarter after the end of the estimation period was chosen to begin the simulation rather than the first quarter because, as indicated in Section III, in the actual use of Model I for forecasting purposes there is a one quarter gap between the end of the estimation period and the beginning of the forecast period. (This procedure thus differs slightly from the procedure discussed in Section III, where there was no gap.)

In computing the actual forecast errors, data on the endogenous variables through 1977II were used. (The preliminary data for 1977III were not used.) This meant that for each variable i, 33 values of d_{it} (as defined in Section II) could be computed for the one-quarter-ahead forecast, 32 values for the two-quarter-ahead forecast, and so on through 26 values for the eight-quarter-ahead forecast. The estimates $\bar{d}_{i1}$ are thus based on 33 observations, the estimates $\bar{d}_{i2}$ on 32 observations, and so on.

The 33 stochastic simulations were performed in the same was as described above for the stochastic simulation with respect to the error terms and coefficient estimates. The only difference in this case is that the simulations were for 8 quarters rather than 13 and the number of trials was 100 rather than 625.[16]

[16] For the eight-quarter simulations each trial takes about 1.0 seconds of computer time, so the total time for the 3300 trials was about 55 minutes. Two points about these computations should be noted. First, subroutine LUDECP (see footnote 11) was unable to factor 26 of the 924 (28 x 33) estimated variance-covariance matrices of the coefficient estimates. This

For four of the six selected variables (real GNP, the GNP deflator, the wage rate, and the money supply) the $\bar{d}_{ik}$ values were computed in percentage terms. To be more precise about what this means, consider the k-quarter-ahead forecasts for variable i. As in Section II, let $\hat{\bar{y}}_{it}$ denote the stochastic-simulation estimate of the k-quarter-ahead expected value of y_{it}, and let $\hat{\sigma}^2_{it}$ denote the stochastic-simulation estimate of the variance of the k-quarter-ahead forecast error. In absolute terms, the actual forecast error is $y_{it} - \hat{\bar{y}}_{it}$, which is denoted $\hat{\varepsilon}_{it}$. In percentage terms, the actual forecast error as a percent of the estimated mean of y_{it} is $\hat{\varepsilon}_{it}/\hat{\bar{y}}_{it}$, and the stochastic-simulation estimate of the standard deviation of the forecast error as a percent of the estimated mean of y_{it} is $\hat{\sigma}_{it}/\hat{\bar{y}}_{it}$. Now, instead of defining d_{it} to be $\hat{\varepsilon}^2_{it} - \hat{\sigma}^2_{it}$, it can be defined to be $(\hat{\varepsilon}_{it}/\hat{\bar{y}}_{it})^2 - (\hat{\sigma}_{it}/\hat{\bar{y}}_{it})^2$. These values of d_{it} can then be used to compute $\bar{d}_{ik}$. For variables that have trends, as the above four do, it seems better to define the degree of misspecification in percentage terms rather than in absolute terms, and so this was done for these four variables.

---

problem appeared to be due to rounding error in the estimation program. In these 26 cases, the last "good" estimate of the variance-covariance matrix was used instead. In other words, if V_i was good for the sample period ending in 1972I and bad for the sample period ending in 1972II, the good estimate was used for both periods. The actual coefficient estimates for the second period were, however, still used.

Second, for a few of the 3300 trials the program that solves the model (by the Gauss-Seidel method) failed. The maximum number of times this happened for a given set of estimates was 15, and the total number of failures was 99. In all, or nearly all, of these cases it is likely that a solution could have been found by fiddling with the solution program. This, however, was not done, and instead the trials that failed were merely not counted. Since many of the trials that failed are likely to have been extreme draws, this procedure probably results in somewhat smaller estimates of the forecast-error variances than would be the case had all the solutions been found. This bias will then be translated into values of $\bar{d}_{ik}$ that are too large. Given the small number of failures, however, it seems unlikely that this bias is very large. The problem of failures was not at all serious for the three stochastic-simulation runs for the 1977IV-1980IV period. No failures occurred for the first run (of 500 trials), and only one each occurred for the other two runs (of 625 trials each).

The results of the misspecification calculations are presented in the following manner in Table 1. First, each number in row d is the square root of the sum of $\bar{d}_{ik}$ and the respective number in row c squared. The numbers in row d are the estimates of the total uncertainty of the forecasts, the estimates being presented in terms of standard errors rather than variances. Each number in row e is then the difference between the respective numbers in rows c and d. The numbers in row e are one way of presenting estimates of the degree of misspecification of the model. Note that these numbers are not the $\bar{d}_{ik}$ values. The $\bar{d}_{ik}$ values are of less interest to present because they are in the units of the variables squared.

Two other tables of results are presented in this paper for reference purposes. The 33 sets of stochastic-simulation estimates of the standard errors (in percentage terms) for real GNP are presented in Table 3. These are the numbers that were used in computing the $\bar{d}_{ik}$ values for real GNP (i.e., they are the $\hat{\sigma}_{it}/\hat{\bar{y}}_{it}$ numbers). The root mean squared errors of the outside-sample forecasts are presented in Table 4. As noted in Section II, RMSEs are often computed for models, although they have no rigorous statistical interpretation.

V. *Estimated Uncertainty for the Model II Forecast*

The procedure followed for the naive model is quite similar to the procedure followed for Model I, and so the discussion of the naive model can be brief. Each of the six equations of the model was treated individually: no attempt was made to estimate and account for the possible correlation of the error terms across equations. The model thus consists of six completely unrelated equations.

The variance of the error term in each equation was estimated as the sum of squared residuals divided by the number of observations. For purposes of the stochastic simulation, the error term in each equation was then assumed to be normally distributed with mean zero and variance as estimated. The coefficient estimates were treated in the same way as they were for Model I. The number of trials for both the simulation with respect to the error terms and the simulation with respect to the error terms and coefficient estimates was 2000.[17] There are no exogenous variables in the naive model, and so no stochastic simulation regarding the exogenous-variable forecasts was needed. For the misspecification estimates, the naive model was also estimated 33 times, and the same periods were used here as were used for Model I. The number of trials for each of the 33 stochastic simulations was 500.[18] The results for the naive model are also presented in Tables 1-4.

VI. The Results

Model I

The results for Model I will be discussed first in this section and then they will be compared to the results for the naive model.

The results in Table 2 should provide some encouragement to model builders. They show that the forecast values computed by setting the error terms equal to zero and solving once are quite close to the forecast values

---

[17]The cost of one trial for the naive model was quite small, and so a large number of trials could be taken.

[18]In one case out of the 33 x 6 estimates, the estimated variance-covariance matrix failed to factor, and this case was skipped for the computations. No other problems were encountered for any of the naive-model computations.

computed by means of stochastic simulation. Although, as noted in Section II, it is well known that the common practice of setting the error terms to zero and solving once produces biased estimates of the true means of the endogenous variables for nonlinear models, this bias does not appear to be very large, at least for Model I.[19] The results in Table 2 thus provide some justification for model builders to continue to do what they have been doing all along.

Consider now Table 1. The results in the a , b , and c rows are fairly self explanatory. As might be expected, the sensitivity of the standard errors of the forecasts to exogenous-variable uncertainty (rows c versus b) is greater for some variables than for others. This sensitivity is small for the unemployment rate, the wage rate, and the bill rate and fairly large for the money supply. Although in most cases these sensitivity differences can be explained, given a knowledge of the structure of the model, this kind of discussion is unnecessary for purposes of this paper. Suffice it to say that I did gain some insights about the model from trying to find explanations for the various sensitivity differences. Estimates of this kind are likely to be of interest to model builders, and they are another example of a useful by-product of the method.

The numbers in the e rows in Table 1 are the estimates of the degree of misspecification of the model. These numbers are fairly small for real GNP, the GNP deflator, and the bill rate. The model does not appear to be very badly misspecified with respect to these variables.

---

[19]Remember, however, that the stochastic-simulation estimates themselves are not quite right in that they are based on a limited number of trials and on only estimated variance-covariance matrices. The results in Table 2 thus do not provide a completely accurate estimate of the bias that results from setting the error terms equal to zero. The conclusion reached here that the bias is small has also been reached by Nagar [18], Sowey [19], Cooper [3], Bianchi, Colzolani, and Corsi [1], and Colzolani and Corsi [2] for their stochastic simulations with respect to the error terms only.

TABLE 1. Estimated Standard Errors of Forecasts

a = uncertainty due to error terms.
b = uncertainty due to error terms and coefficient estimates.
c = uncertainty due to error terms, coefficient estimates, and exogenous-variable forecasts.
d = uncertainty due to error terms, coefficient estimates, exogenous-variable forecasts, and possible misspecifications of the model.
e = estimate of the degree of misspecification of the model ($e = d - c$) .

Forecast Period = 1977IV-1980IV.

Model I = model in [8].
Model II = naive model. For the naive model there are no exogenous variables, so $c = b$ for this model.

For the unemployment rate and the bill rate, the errors are in the natural units of the variables. For the other variables, the errors are expressed as percentages of the forecast means (in percentage points).

	1977IV	*1978I*	*1978II*	*1978III*	*1978IV*	*1979I*	*1979II*	*1979III*	*1979IV*	*1980I*	*1980II*	*1980III*	*1980IV*
Model I. Real GNP													
a	0.64	0.92	1.13	1.21	1.30	1.31	1.34	1.36	1.39	1.39	1.37	1.39	1.42
b	0.65	0.98	1.24	1.45	1.58	1.65	1.71	1.71	1.77	1.77	1.77	1.81	1.82
c	0.75	1.09	1.40	1.57	1.77	1.90	1.99	2.11	2.11	2.14	2.17	2.21	2.15
d	0.91	1.47	1.87	2.20	2.56	2.70	2.51	2.11					
e	(0.16)	(0.38)	(0.47)	(0.63)	(0.79)	(0.80)	(0.52)	(0.00)					
Model II. Real GNP													
a	0.63	1.03	1.34	1.62	1.81	1.89	1.95	2.03	2.04	2.05	2.07	2.06	2.05
b,c	0.67	1.13	1.51	1.87	2.19	2.38	2.51	2.60	2.68	2.73	2.80	2.87	2.94
d	1.12	1.96	2.71	3.43	4.00	4.36	4.66	4.85					
e	(0.45)	(0.83)	(1.20)	(1.56)	(2.19)	(1.98)	(2.15)	(2.25)					
Model I. GNP Deflator													
a	0.28	0.40	0.50	0.60	0.69	0.78	0.87	0.94	1.03	1.08	1.14	1.21	1.22
b	0.33	0.50	0.68	0.88	1.05	1.21	1.39	1.58	1.72	1.88	2.05	2.22	2.36
c	0.48	0.71	0.94	1.15	1.34	1.53	1.75	1.93	2.19	2.41	2.56	2.69	2.82
d	0.59	0.95	1.37	1.73	2.09	2.42	2.86	3.32					
e	(0.11)	(0.24)	(0.43)	(0.58)	(0.75)	(0.89)	(1.11)	(1.39)					
Model II. GNP Deflator													
a	0.19	0.34	0.49	0.66	0.84	0.98	1.11	1.20	1.26	1.31	1.33	1.34	1.33
b,c	0.25	0.49	0.78	1.13	1.52	1.92	2.33	2.69	3.02	3.30	3.53	3.74	3.91
d	0.45	0.95	1.55	2.31	3.26	4.28	5.38	6.51					
e	(0.20)	(0.46)	(0.77)	(1.18)	(1.74)	(2.36)	(3.05)	(3.82)					
Model I. Unemployment Rate (units of percentage points)													
a	0.29	0.46	0.58	0.66	0.73	0.80	0.85	0.89	0.90	0.95	0.94	0.99	1.06
b	0.35	0.60	0.78	0.94	1.08	1.17	1.24	1.32	1.37	1.48	1.58	1.65	1.70
c	0.35	0.60	0.81	0.99	1.11	1.26	1.37	1.49	1.61	1.70	1.80	1.89	1.94
d	0.33	0.54	0.63	0.70	0.74	0.84	0.78	0.67					
e	(-0.02)	(-0.06)	(-0.18)	(-0.29)	(-0.37)	(-0.42)	(-0.59)	(-0.82)					
Model II. Unemployment Rate (units of percentage points)													
a	0.29	0.56	0.79	0.95	1.04	1.08	1.11	1.13	1.14	1.15	1.15	1.16	1.17
b,c	0.32	0.66	0.94	1.12	1.25	1.33	1.43	1.52	1.59	1.64	1.69	1.72	1.77
d	0.39	0.84	1.24	1.59	1.86	2.04	2.23	2.37					
e	(0.07)	(0.18)	(0.30)	(0.47)	(0.61)	(0.71)	(0.80)	(0.85)					

TABLE 1 (cont.)

	1977IV	1978I	1978II	1978III	1978IV	1979I	1979II	1979III	1979IV	1980I	1980II	1980III	1980IV
Model I. Wage Rate													
a	0.25	0.47	0.66	0.84	0.98	1.11	1.19	1.26	1.30	1.35	1.41	1.46	1.50
b	0.27	0.52	0.75	0.97	1.16	1.37	1.55	1.71	1.89	2.05	2.22	2.38	2.54
c	0.28	0.52	0.73	0.96	1.15	1.35	1.52	1.71	1.92	2.10	2.25	2.39	2.53
d	0.47	0.97	1.54	2.18	2.85	3.60	4.41	5.26					
e	(0.20)	(0.45)	(0.79)	(1.22)	(1.70)	(2.25)	(2.89)	(3.55)					
Model II. Wage Rate													
a	0.14	0.23	0.32	0.43	0.54	0.65	0.76	0.85	0.94	1.02	1.09	1.15	1.20
b,c	0.17	0.30	0.46	0.66	0.89	1.14	1.41	1.68	1.95	2.24	2.53	2.83	3.12
d	0.37	0.63	0.92	1.32	1.83	2.29	2.79	3.21					
e	(0.20)	(0.33)	(0.46)	(0.66)	(0.94)	(1.15)	(1.38)	(1.53)					
Model I. Bill Rate (units of percentage points)													
a	0.43	0.72	0.85	0.91	0.95	1.00	1.02	1.05	1.04	1.04	1.02	1.02	1.05
b	0.45	0.70	0.89	1.06	1.16	1.23	1.31	1.32	1.33	1.38	1.39	1.42	1.47
c	0.46	0.74	0.90	1.04	1.19	1.24	1.32	1.39	1.48	1.52	1.54	1.52	1.54
d	0.61	1.04	1.09	1.14	1.23	1.17	1.01	0.79					
e	(0.15)	(0.30)	(0.19)	(0.10)	(0.04)	(-0.07)	(-0.31)	(-0.60)					
Model II. Bill Rate (units of percentage points)													
a	0.45	0.71	0.80	0.86	0.90	0.93	0.94	0.94	0.95	0.96	0.98	1.01	1.01
b,c	0.49	0.80	0.95	1.04	1.15	1.23	1.27	1.28	1.25	1.22	1.22	1.23	1.21
d	0.73	1.18	1.38	1.51	1.62	1.71	1.79	1.90					
e	(0.24)	(0.38)	(0.43)	(0.47)	(0.47)	(0.48)	(0.52)	(0.62)					
Model I. Money Supply													
a	0.85	1.14	1.38	1.54	1.66	1.81	1.84	1.86	1.99	2.06	2.16	2.24	2.26
b	0.86	1.28	1.65	1.98	2.20	2.47	2.80	2.97	3.22	3.45	3.67	3.91	4.10
c	0.94	1.47	1.79	2.18	2.58	3.03	3.43	3.80	4.21	4.58	4.96	5.34	5.74
d	1.50	2.41	3.26	4.31	5.36	6.39	7.54	8.68					
e	(0.56)	(0.94)	(1.47)	(2.13)	(2.78)	(3.36)	(4.11)	(4.88)					
Model II. Money Supply													
a	0.56	0.67	0.81	0.90	0.96	1.03	1.11	1.17	1.24	1.30	1.33	1.38	1.41
b,c	0.64	0.80	1.00	1.21	1.35	1.50	1.70	1.85	2.04	2.23	2.39	2.57	2.76
d	1.42	1.64	1.90	2.28	2.58	2.94	3.35	3.74					
e	(0.78)	(0.84)	(0.90)	(1.07)	(1.23)	(1.44)	(1.65)	(1.89)					

TABLE 2. Estimated Forecast Means

0 = error terms set equal to zero (no stochastic simulation).
a = stochastic simulation with respect to error terms only.
b = stochastic simulation with respect to error terms and coefficient estimates.
c = stochastic simulation with respect to error terms, coefficient estimates, and exogenous variables.

Model I = model in [8].
Model II = naive model.

	1977IV	1978I	1978II	1978III	1978IV	1979I	1979II	1979III	1979IV	1980I	1980II	1980III	1980IV
Model I. Real GNP (billions of 1972 dollars)													
0	1356.4	1376.2	1396.1	1415.5	1431.3	1446.9	1461.2	1474.6	1485.7	1497.4	1509.1	1520.8	1531.3
a	1355.9	1375.5	1394.7	1414.3	1429.9	1445.8	1460.0	1474.5	1485.7	1497.2	1508.3	1519.7	1530.5
b	1357.0	1376.4	1395.5	1413.8	1427.9	1443.2	1457.1	1470.9	1482.2	1494.0	1506.6	1518.3	1529.2
c	1356.6	1375.9	1395.2	1414.7	1429.6	1444.7	1458.7	1471.7	1483.0	1494.6	1506.0	1518.1	1529.5
Model II. Real GNP (billions of 1972 dollars)													
0	1356.4	1368.4	1373.5	1376.7	1382.7	1388.2	1393.0	1400.0	1407.7	1414.5	1421.3	1428.8	1435.8
a	1356.5	1368.5	1373.4	1376.4	1382.4	1387.6	1392.5	1399.4	1407.5	1414.2	1421.2	1429.0	1436.1
b	1356.6	1368.6	1373.9	1377.7	1384.2	1389.8	1394.6	1402.3	1410.3	1417.2	1424.3	1432.0	1439.0
Model I. GNP Deflator (1972 = 1.0)													
0	1.4406	1.4574	1.4750	1.4935	1.5148	1.5341	1.5538	1.5737	1.5969	1.6169	1.6370	1.6572	1.6803
a	1.4405	1.4578	1.4751	1.4934	1.5145	1.5338	1.5534	1.5735	1.5972	1.6178	1.6386	1.6590	1.6828
b	1.4403	1.4577	1.4756	1.4944	1.5162	1.5361	1.5567	1.5772	1.6006	1.6211	1.6415	1.6623	1.6864
c	1.4409	1.4580	1.4753	1.4942	1.5157	1.5353	1.5551	1.5755	1.5995	1.6202	1.6408	1.6614	1.6854
Model II. GNP Deflator (1972 = 1.0)													
0	1.4501	1.4822	1.5168	1.5556	1.5978	1.6426	1.6892	1.7358	1.7824	1.8286	1.8739	1.9186	1.9630
a	1.4500	1.4821	1.5168	1.5557	1.5979	1.6428	1.6895	1.7362	1.7828	1.8291	1.8745	1.9193	1.9638
b	1.4501	1.4822	1.5168	1.5556	1.5980	1.6429	1.6897	1.7364	1.7832	1.8292	1.8744	1.9188	1.9628
Model I. Unemployment Rate (percentage points)													
0	7.10	7.07	6.94	6.80	6.74	6.72	6.72	6.75	6.82	6.91	6.99	7.07	7.17
a	7.10	7.10	6.98	6.83	6.77	6.75	6.76	6.78	6.81	6.89	6.97	7.07	7.19
b	7.11	7.08	6.93	6.77	6.73	6.73	6.72	6.73	6.77	6.85	6.93	7.00	7.07
c	7.11	7.08	6.95	6.80	6.72	6.74	6.71	6.73	6.81	6.89	6.99	7.09	7.18
Model II. Unemployment Rate (percentage points)													
0	7.21	7.59	7.74	7.64	7.50	7.41	7.41	7.47	7.47	7.41	7.36	7.33	7.35
a	7.21	7.58	7.73	7.62	7.48	7.38	7.39	7.45	7.47	7.41	7.36	7.32	7.33
b	7.21	7.60	7.77	7.68	7.55	7.47	7.49	7.56	7.57	7.51	7.47	7.46	7.48

TABLE 2 (cont.)

	1977IV	1978I	1978II	1978III	1978IV	1979I	1979II	1979III	1979IV	1980I	1980II	1980III	1980IV
Model I. Wage Rate (1967 = 100.0)													
0	203.7	207.6	211.6	215.6	219.7	223.8	228.0	232.2	236.5	240.7	245.0	249.4	253.7
a	203.7	207.6	211.6	215.6	219.7	223.7	227.9	232.2	236.5	240.8	245.3	249.7	254.1
b	203.7	207.6	211.6	215.7	219.8	223.9	228.2	232.4	236.7	241.0	245.4	249.8	254.2
c	203.7	207.6	211.6	215.6	219.7	223.8	228.1	232.4	236.6	241.0	245.3	249.7	254.2
Model II. Wage Rate (1967 = 100.0)													
0	203.7	207.6	211.6	215.8	220.1	224.6	229.1	233.9	238.7	243.7	248.8	254.0	259.3
a	203.7	207.6	211.6	215.8	220.1	224.6	229.2	233.9	238.7	243.7	248.8	254.0	259.3
b	203.7	207.6	211.6	215.8	220.0	224.5	229.1	233.8	238.6	243.5	248.5	253.7	259.0
Model I. Bill Rate (percentage points)													
0	5.80	6.07	6.38	6.69	6.93	7.15	7.34	7.50	7.61	7.71	7.79	7.87	7.92
a	5.82	6.07	6.33	6.63	6.84	7.10	7.30	7.46	7.59	7.70	7.77	7.83	7.85
b	5.82	6.11	6.40	6.69	6.90	7.12	7.27	7.45	7.58	7.72	7.87	7.95	7.99
c	5.80	6.07	6.39	6.68	6.91	7.09	7.29	7.46	7.58	7.70	7.82	7.91	7.97
Model II. Bill Rate (percentage points)													
0	6.03	6.51	7.05	7.53	7.85	7.99	8.04	8.01	7.86	7.66	7.45	7.27	7.12
a	6.03	6.49	7.03	7.51	7.83	7.96	8.02	8.01	7.88	7.66	7.46	7.27	7.12
b	6.04	6.50	7.01	7.48	7.82	7.98	8.02	7.96	7.82	7.60	7.37	7.19	7.04
Model I. Money Supply (billions of current dollars)													
0	359.2	367.0	374.7	382.6	390.5	398.5	406.5	414.6	422.8	431.1	439.4	447.7	456.2
a	359.2	367.1	374.7	382.5	390.3	398.3	406.6	414.9	423.2	431.3	439.6	447.9	456.5
b	359.2	366.8	374.4	382.2	390.0	398.1	406.2	414.2	422.6	431.1	439.3	447.6	456.3
c	359.2	367.0	374.5	382.5	390.4	398.2	406.3	414.5	422.4	430.6	439.1	447.3	456.0
Model II. Money Supply (billions of current dollars)													
0	354.7	360.7	365.2	369.3	375.4	380.4	385.4	391.0	396.0	401.3	406.9	412.2	417.8
a	354.6	360.6	365.1	369.3	375.4	380.4	385.4	390.9	395.9	401.3	406.9	412.2	417.7
b	354.7	360.7	365.4	369.5	375.7	380.8	385.8	391.4	396.4	401.7	407.5	412.9	418.6

TABLE 3. Standard Errors Estimated by Stochastic Simulation for Real GNP for Use in Computing the $\bar{d}_{ik}$ Values

The errors are expressed as percentages of the forecast means (in percentage points).

End of Estimation	*MODEL I*								*MODEL II*							
	Number of Quarters Ahead															
Period	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1968IV	0.66	1.10	1.37	1.71	2.12	2.57	3.42	3.98	0.59	1.04	1.39	1.66	1.89	2.09	2.24	2.42
1969I	0.66	0.99	1.40	1.74	2.02	2.44	2.95	3.22	0.62	1.03	1.39	1.70	1.97	2.16	2.37	2.54
II	0.70	1.17	1.53	1.77	2.19	2.51	2.78	2.95	0.61	1.02	1.38	1.72	1.96	2.16	2.31	2.43
III	0.60	1.01	1.36	1.66	2.03	2.42	2.75	2.85	0.64	1.08	1.47	1.82	2.11	2.26	2.36	2.44
IV	0.66	1.15	1.61	1.89	2.18	2.41	2.67	2.79	0.64	1.16	1.61	2.06	2.41	2.68	2.89	3.05
1970I	0.70	1.09	1.46	1.75	2.11	2.17	2.42	2.68	0.63	1.18	1.66	2.07	2.38	2.64	2.82	2.98
II	0.76	1.21	1.53	1.88	2.18	2.43	2.70	2.88	0.63	1.20	1.69	2.18	2.57	2.80	2.95	3.12
III	0.84	1.21	1.51	1.81	1.97	2.22	2.52	2.79	0.64	1.14	1.65	2.09	2.42	2.63	2.80	2.98
IV	0.69	1.14	1.42	1.60	1.79	1.97	2.10	2.39	0.74	1.27	1.74	2.20	2.56	2.88	3.14	3.46
1971I	0.82	1.30	1.54	1.77	1.89	1.95	2.02	2.07	0.75	1.14	1.51	1.83	2.18	2.27	2.41	2.58
II	0.76	1.10	1.31	1.60	1.86	1.97	2.19	2.48	0.80	1.04	1.40	1.68	1.88	2.00	2.10	2.24
III	0.62	1.06	1.31	1.60	1.83	2.05	2.28	2.59	0.68	1.05	1.39	1.62	1.80	1.90	2.00	2.12
IV	0.71	0.92	1.15	1.28	1.43	1.64	2.01	2.34	0.70	0.97	1.30	1.55	1.81	1.91	2.01	2.16
1972I	0.76	1.11	1.35	1.54	1.69	1.90	2.27	2.86	0.71	1.03	1.36	1.62	1.91	2.07	2.21	2.32
II	0.64	1.01	1.28	1.44	1.70	2.02	2.53	3.11	0.65	1.04	1.44	1.79	2.11	2.33	2.52	2.67
III	0.66	1.02	1.28	1.40	1.74	2.20	2.70	3.16	0.60	0.97	1.30	1.65	1.91	2.12	2.31	2.45
IV	0.63	1.04	1.33	1.66	2.14	2.33	2.68	3.28	0.62	1.04	1.43	1.85	2.24	2.49	2.67	2.82
1973I	0.66	1.11	1.54	1.86	2.21	2.63	3.14	3.77	0.64	1.11	1.55	1.98	2.38	2.69	2.92	3.13
II	0.77	1.22	1.64	2.04	2.38	2.74	3.26	3.82	0.66	1.03	1.41	1.74	2.07	2.21	2.35	2.53
III	0.73	1.29	1.78	2.05	2.38	2.82	3.21	3.66	0.60	1.05	1.42	1.78	2.09	2.35	2.53	2.69
IV	0.69	1.19	1.58	2.01	2.41	2.84	3.07	3.55	0.69	1.07	1.43	1.79	2.06	2.27	2.49	2.66
1974I	0.65	1.05	1.52	2.06	2.78	3.41	4.34	5.95	0.69	1.13	1.53	1.95	2.27	2.49	2.66	2.79
II	0.66	1.07	1.60	2.02	2.51	3.24	4.35	5.57	0.69	1.12	1.59	2.02	2.37	2.58	2.72	2.79
III	0.71	1.39	1.88	2.24	2.91	3.67	4.30	5.24	0.70	1.24	1.73	2.34	2.85	3.14	3.32	3.37
IV	0.92	1.59	1.98	2.37	2.85	3.37	4.17	5.16	0.78	1.52	2.26	3.03	3.67	4.16	4.51	4.67
1975I	0.80	1.41	2.02	2.55	2.91	3.38	3.92	4.47	0.86	1.51	2.26	3.02	3.78	4.42	4.87	5.30
II	0.76	1.24	1.71	2.23	2.65	3.13	3.57		0.85	1.44	1.90	2.49	2.96	3.13	3.19	
III	0.91	1.60	2.12	2.39	2.75	2.94			0.85	1.48	2.01	2.51	2.85	2.95		
IV	0.68	1.11	1.64	2.09	2.46				0.86	1.48	1.87	2.25	2.62			
1976I	0.65	1.00	1.45	1.82					0.83	1.50	1.91	2.29				
II	0.75	1.29	1.57						0.77	1.24	1.59					
III	0.72	1.08							0.76	1.23						
IV	0.67								0.72							

TABLE 4. Root Mean Squared Errors of Outside-Sample Forecasts

		Number of Quarters Ahead							
Variable	*Model*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Real GNP (percent)	*I*	0.88	1.54	1.99	2.43	2.91	3.23	3.42	3.57
	II	1.15	1.99	2.77	3.53	4.12	4.50	4.83	5.05
GNP Deflator (percent)	*I*	0.56	1.01	1.52	2.00	2.50	2.94	3.44	3.94
	II	0.46	0.97	1.58	2.35	3.30	4.35	5.50	6.72
Unemployment Rate (percentage points)	*I*	0.37	0.58	0.68	0.73	0.80	0.86	0.85	0.82
	II	0.37	0.78	1.16	1.53	1.80	1.97	2.12	2.23
Wage Rate (percent)	*I*	0.53	1.08	1.74	2.46	3.24	4.08	4.98	5.90
	II	0.38	0.66	0.97	1.42	1.95	2.54	3.18	3.82
Bill Rate (percentage points)	*I*	0.65	1.11	1.26	1.38	1.53	1.61	1.57	1.56
	II	0.72	1.19	1.39	1.52	1.63	1.70	1.76	1.87
Money Supply (percent)	*I*	1.40	2.25	3.13	4.22	5.32	6.39	7.62	8.88
	II	1.44	1.72	2.07	2.49	2.87	3.29	3.71	4.19

Note: For a given length of forecast, let $\hat{\varepsilon}_{it} = y_{it} - \hat{\bar{y}}_{it}$ be the actual forecast error for variable i for quarter t, where y_{it} is the actual value and $\hat{\bar{y}}_{it}$ is the expected value estimated by stochastic simulation. (Stochastic simulation in this case is with respect to the error terms and coefficient estimates only. The exogenous-variable values are the actual values.) Then the RMSE for this variable and length of forecast is either $\sqrt{\frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_{it}^2}$ or $\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\varepsilon}_{it}/\hat{\bar{y}}_{it})^2}$, where T is the number of observations. The former pertains to the unemployment rate and the bill rate, and the latter pertains to the other variables. T is 33 for the one-quarter-ahead forecasts, 32 for the two-quarter-ahead forecasts, and so on.

For the unemployment rate the numbers are negative and grow progressively larger in absolute value. For the eight-quarter-ahead forecast the estimate of the degree of misspecification is -0.82 percentage points, which is (in absolute value) 55.0 percent of the stochastic-simulation estimate of 1.49 percentage points of the standard error of the forecast in row *c* .

The misspecification estimates are highest for the wage rate and the money supply. The eight-quarter-ahead estimate of the degree of misspecification for the wage rate is 3.55 percent, which is slightly more than double the stochastic-simulation estimate of 1.71 percent in row *c* . For the money supply the eight-quarter-ahead estimate is 4.88 percent, which compares to the stochastic-simulation estimate of 3.80 percent in row *c* . The model appears to be clearly misspecified with respect to these two variables.

The numbers in the *d* rows are the estimates of the total uncertainty of the forecasts. A brief summary of them is as follows. For the four-quarter-ahead forecasts, the estimated standard errors are 2.20 percent (31.1 billion dollars)[20] for real GNP, 1.73 percent for the GNP deflator, 0.70 percentage points for the unemployment rate, 2.18 percent for the wage rate, 1.14 percentage points for the bill rate, and 4.31 percent (16.5 billion dollars) for the money supply. For the eight-quarter-ahead forecasts, the estimated standard errors are 2.11 percent (31.1 billion dollars) for real GNP, 3.32 percent for the GNP deflator, 0.67 percentage points for the unemployment rate, 5.26 percent for the wage rate, 0.79 percentage points for the bill rate, and 8.68 percent (36.0 billion dollars) for the money supply.

---

[20] Any dollar figure used in this section has been obtained by multiplying the particular percent figure in Table 1 by the relevant number in the *c* rows in Table 2.

Model I versus Model II

The results for Model II, the naive model, are also presented in Tables 1 and 2. Note first in Table 2 that the two models are projecting quite different paths for real GNP and the GNP deflator. By 1980IV the forecasts of real GNP differ by 90.5 billion dollars and the forecasts of the GNP deflator differ by 16.5 percent.[21] Relative to Model I, the naive model is predicting less real GNP and more inflation.

From the results in the *e* rows in Table 1 it can be seen that the naive model appears to be misspecified for all the variables. With respect to the estimates of the total uncertainty of the forecasts in the *d* rows, the naive model is less accurate than Model I for real GNP, the GNP deflator, the unemployment rate, and the bill rate, and it is more accurate for the wage rate and the money supply. For the eight-quarter-ahead forecasts, the differences in the estimated standard errors are 2.74 percent for real GNP, 3.19 percent for the GNP deflator, 1.70 percentage points for the unemployment rate, -2.05 percent for the wage rate, 1.11 percentage points for the bill rate, and -4.94 percent for the money supply.

Given my wage rate, I would conclude from the results in Table 1 that Model I is enough of an improvement over the naive model to justify the time that I have so far spent developing and working on it. The differences in the standard errors for real GNP, the GNP deflator, the unemployment rate, and the bill rate are substantial. It is, of course, somewhat embarrassing that Model I is less accurate with respect to the fore-

[21]90.5 billion dollars = Model I row *c* estimate - Model II row *b* estimate, and 0.165 = Model II row *b* estimate/Model I row *c* estimate - 1.0.

casts of the wage rate and the money supply than the naive model. There is not too much that can be said about this except that I was aware before, and even more so now, that the wage-rate equation and one of the demand-for-money equations are two of the weakest equations in Model I, weakest in the sense that the coefficient estimates of these two equations tend to change more as the model is reestimated on the basis of new data than do the coefficient estimates of most of the other equations. There are clearly grounds for further work on these two equations.

Two further points about the negative results for the wage rate and money supply should be noted. First, there is some evidence that indicates that the demand-for-money equations in Model I are more accurate than other demand-for-money equations. As discussed in footnote 5 in Fair [9], the demand-for-money equations in Model I appear to be considerably more accurate for the 1973I-1967I period than the demand-for-money equation in the MPS model. The problems noted here regarding the demand-for-money equation in Model I are thus probably not unique to Model I.

Second, it is interesting to note that the problems with the wage-rate and demand-for money equations do not appear to have a serious effect on the overall model. I think, however, that this is more the case for the demand-for-money equation than it is for the wage-rate equation. From other experiments that I have done, it seems quite likely that the overall accuracy of the model would be considerably improved if a better wage-rate equation were found, especially with respect to the accuracy of the forecasts of the GNP deflator. If, on the other hand, a more accurate demand-for-money equation were found, this would probably not have much effect on the overall accuracy of the model. The feedbacks from the errors in predicting the demand for money to other errors in the model are not very large in the version of the model that includes the equation explaining the behavior of the Federal Reserve.

Consider finally the RMSE results in Table 4. As noted in Section IV, RMSEs are commonly used to judge a model's accuracy. Using the RMSEs in Table 4 to compare the accuracy of Model I to that of Model II ignores, however, the uncertainty due to the exogenous-variable forecasts and the fact that the forecast-error variances are not constant across time. Since a comparison of the accuracy of the two models on the basis of the estimated standard errors in the d rows in Table 1 does not ignore these two problems, it seems much better to use the results in Table 1 to compare the two models than the results in Table 4. Comparing the numbers in the d rows in Table 1 to the respective numbers in Table 4, it can be seen that there are some sizeable differences, although the overall ranking of the accuracy of the two models by variable is the same.

VII. Summary and Conclusion

The primary purpose of this paper has been to propose a method for estimating the combined uncertainty of an econometric-model forecast from the error terms, the coefficient estimates, the exogenous-variable forecasts, and the possible misspecification of the model. The method accounts for the fact that the forecast-error variances are not constant across time, and it provides for each variable and length of forecast a quantitative estimate of the degree of misspecification of a model.

Estimating the uncertainty from the first three sources is not very expensive. It merely requires one stochastic-simulation over the forecast period. This stochastic simulation is a straightforward procedure except for the treatment of the exogenous-variable forecasts. Some estimate or assumption about the uncertainty of the latter must be made prior to the stochastic simulation. Estimating the uncertainty from the possible misspecification of the model is much more costly, since it requires successive reestimation and stochastic simulation of the model. These calculations, however, are not forecast specific, and so they need not be done each time a new forecast is made. The proposed estimates of the degree of misspecification of a model are clearly only approximations, since they rest on the fairly restrictive assumption that the degree of misspecification of a model by variable and length of forecast is constant across time.

The main conclusions from the empirical work are the following:

1. The common practice of solving a model by setting the error terms equal to zero does not appear to lead to serious biases in the estimates of the expected values of the endogenous variables. (Table 2)
2. The model in Fair [4], Model I, appears to be seriously misspecified for the wage rate and the money supply. The naive model, Model II, appears to be misspecified for all the variables. (Table 1)
3. Model I is more accurate than Model II for real GNP, the GNP deflator, the unemployment rate, and the bill rate. It is less accurate for the wage rate and the money supply. (Table 1)
4. There are some sizeable differences between the "incorrect" estimates of uncertainty in Table 4 (the RMSE estimates) and the "correct" estimates in the d rows in Table 1, although the overall ranking of the accuracy of the two models by variable is the same in the two tables.

It is hoped that the method proposed in this paper will become used by model builders. Results like those presented in Table 1 for other models would be of considerable interest and would help in evaluating their relative strengths and weaknesses. The way that is proposed in this study for estimating the degree of misspecification of a model does require that a model be reestimated each quarter, which is not yet the common practice among model builders. It would be possible to modify the method proposed here to accommodate a model that was, say, only reestimated once a year, but this would require a stronger "constancy" assumption than the one needed here. It is not now, however, very expensive to estimate a model of even 100 stochastic equations, and so the requirement that a model be reestimated once a quarter does not seem too severe.

The method proposed in this paper is not relevant for forecasts from models that are subjectively adjusted. Even for subjectively-adjusted models, however, the following procedure could be followed. (1) Treat the model mechanically and perform the calculations necessary for results like those in Table 1. For people who are interested in the model qua model, this would be useful information. (2) Over a period of a few years compile an ex ante forecasting record for both the model used mechanically and the model used subjectively. Let e^{m}_{ikt} denote the error of the k-quarter-ahead forecast of variable i for quarter t from the model used mechanically (the forecast starting at the beginning of quarter $t-k$) and let e^{s}_{ikt} denote the similar error for the model used subjectively. Let δ_{ikt} be the difference in the errors squared: $\delta_{ikt} = (e^{m}_{ikt})^2 - (e^{s}_{ikt})^2$. After say, 12 values of δ_{ikt} have been compiled, take the average of these values. Denote this average as $\overline{\delta}_{ik}$. $\overline{\delta}_{ik}$ will be positive if subjectively adjusting the model has on average improved its forecasting accuracy. (3) If it is assumed that the degree to which subjectively adjusting a model improves its forecasting accuracy with respect to a given variable and length of forecast is constant across time, then the $\overline{\delta}_{ik}$ values can be subtracted in the appropriate way from the numbers in the d rows in

Table 1 to get a final estimate of the uncertainty of the forecasts from the subjectively-adjusted model.[22]

The constancy assumption in (3) is, of course, much stronger than the constancy assumption needed for the results in Table 1, but the $\bar{\delta}_{ik}$ values would at least be rough approximations of the degree to which subjectively adjusting a model improves its forecasting accuracy. Of more scientific interest, however, would be the mechanical results themselves, for only by observing results like those in Table 1 for models used mechanically can one hope to learn about the models in ways that are useful for further scientific research.

[22]The appropriate way would be to subtract $\bar{\delta}_{ik}$ from the square of the respective number in row d of Table 1 and then to take the square root of this difference. This number would then be the final estimate of the standard error of the forecast from the subjectively-adjusted model.

REFERENCES

[1] Bianchi, C., G. Calzolari, and P. Corsi, "Divergences in the Results of Stochastic and Deterministic Simulation of an Italian Non Linear Econometric Model," in L. Dekker (ed.), *Simulation of Systems* (Amsterdam: North-Holland Publishing Co., 1976).

[2] Calzolari, Giorgio, and Paolo Corsi, "Stochastic Simulation as a Validation Tool for Econometric Models," paper presented at IIASA Seminar, Laxenburg, Vienna, September 13-15, 1977.

[3] Cooper, J. Phillip, *Development of the Monetary Sector, Prediction and Policy Analysis in the FRB-MIT-Penn Model* (Lexington: D.C. Heath & Co., 1974).

[4] Cooper, J. Phillip, and Stanley Fischer, "Stochastic Simulation of Monetary Rules in Two Macroeconometric Models," *Journal of the American Statistical Association,* LXVII (December 1972), 750-760.

[5] Cooper, J. Phillip, and Stanley Fischer, "Monetary and Fiscal Policy in the Fully Stochastic St. Louis Econometric Model," *Journal of Money, Credit and Banking,* VI (February 1974), 1-22.

[6] Evans, Michael K., Lawrence R. Klein, and Mitsuo Saito, "Short-Run Prediction and Long-Run Simulation of the Wharton Model," in [16].

[7] Fair, Ray C., "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors," *Econometrica*, XXXVIII (May 1970), 507-516.

[8] Fair, Ray C., *A Model of Macroeconomic Activity. Volume II: The Empirical Model* (Cambridge: Ballinger Publishing Co., 1976).

[9] Fair, Ray C., "The Sensitivity of Fiscal-Policy Effects to Assumptions about the Behavior of the Federal Reserve," *Econometrica*, forthcoming 1978.

[10] Fair, Ray C., "The Fair Model as of July 1, 1977," mimeo.

[11] Fisher, Franklin M., Paul H. Cootner, and Martin Neil Baily, "An Econometric Model of the World Copper Industry," *The Bell Journal*, III (Autumn 1972), 568-609.

[12] Fromm, Gary, Lawrence R. Klein, and George R. Schink, "Short- and Long-Term Simulations with the Brookings Model," in [16].

[13] Garbade, Kenneth D., *Discretionary Control of Aggregate Economic Activity* (Lexington: D. C. Heath & Co., 1975).

[14] Green, George R., Maurice Liebenberg, and Albert A. Hirsch, "Short- and Long-Term Simulations with the OBE Econometric Model," in [16].

[15] Hammersley, J. M., and D. C. Handscomb, *Monte Carlo Methods* (London: Methuen & Co., 1974).

[16] Hickman, Bert G. (ed.), *Econometric Models of Cyclical Behavior: Volume I* (New York: Columbia University Press, 1972).

[17] Howrey, E. Philip, and Harry H. Kelejian, "Simulation versus Analytical Solutions: The Case of Econometric Models," in Thomas H. Naylor (ed.), *Computer Simulation Experiments with Models of Economic Systems* (New York: John Wiley, 1971).

[18] Nagar, A. L., "Stochastic Simulation of the Brookings Econometric Model," in James S. Duesenberry, Gary Fromm, Lawrence R. Klein, and Edwin Kuh (eds.), *The Brookings Model: Some Further Results* (Chicago: Rand McNally & Co., 1969).

[19] Sowey, Eric R., "Stochastic Simulation of Macroeconomic Models: Methodology and Interpretation," in Alan A. Powell and Ross A. Williams (eds.), *Econometric Studies of Macro and Monetary Relations* (Amsterdam: North-Holland Publishing Co., 1973).